# Chiral Bosonization of $U_A(1)$-Currents and the Energy-Momentum Tensor in Quantum Chromodynamics [1,2]

Andrianov A.A., Andrianov V.A. ,Yudichev V.L.

*V.A. Fock Department of Theoretical Physics, St.-Petersburg State University, 198504 St.-Petersburg, Russia*

*Dedicated to Memory of Victor N. Popov*



# 1 Introduction.

In the various fields of physics, in particular, in description of quantum phenomena at low energies and temperatures, there appears the problem of extraction of essential (collective) variables and construction of a relevant effective lagrangian [1]. Our interest to these problems had arisen from the numerous discussions with Victor N. Popov, whose experience and contribution to this domain helped us to develop the method of the low-energy bosonization [2] and apply it to the theory of strong interactions — Quantum Chromodynamics .

Chiral lagrangians for the light $SU(3)$ pseudoscalar mesons parameterize their strong and electroweak interactions by means of more than ten constants [3], which contain information about the dynamics of the quark and gluon interaction — Quantum Chromodynamics (QCD). Most of the constants can be evaluated from the analysis of experimental data, thereby there appears a possibility to examine QCD at low energies. For the calculation of the structure constants of the chiral lagrangian the low-energy QCD bosonization method has been developed [2]. This method gives good numerical results for the $SU(3)$ quark current matrix elements [4].

Recently, the necessity to extend the $SU(3)_F$-chiral lagrangian appears to describe $U_A(1)$ current matrix elements [5], in particular, for the pseudoscalar gluon density, and also the matrix elements of the energy-momentum tensor [6] , amonge them,for the scalar gluon density. These matrix elements characterize some decays of heavy quarkonium ($\bar{c}c, \bar{b}b$ and etc. ) [5] and also Higgs bosons, which occur via so-called vacuum channel. Thus, it is interesting to apply the low-energy QCD-bosonization method for the construction of the generalized chiral lagrangian in the presence of external $U_A(1)$-fields and in the background metric $g_{\mu\nu}(x)$.

The main goal of this work is to elaborate the parameterization of the extended chiral lagrangian by means of the low-energy QCD-bosonization method, in the model [2], and derivation of the relations between the chiral coupling constants in the limit of large-$N_c$, number of colors ( generalized Zweig rules ).

[1]Recently the new interest arised [14] to the chiral coefficients $L_{11}, L_{12}, L_{13}$ in the low-energy ChPT lagrangian describing the pseudoscalar meson coupling to gravity and/or light singlet dilatons. In 90ties we estimated these coefficients within the Chiral and Conformal Bosonization Model [8, 4] but published the results in the journal [12] not easily accessible for the physics community. We put now our old paper to the E-archives.

[2]This is a translation of the article published in "Zapiski nauch. sem. POMI" (Proc. Steklov Math. Inst.,St.Petersburg branch) v.224/13 (1995) 68; Engl.transl.[12]; its short version appeared in [13].

In the section 2 we briefly explain the scheme of the low-energy QCD-bosonization, display the structure of the $SU(3)_F$-chiral lagrangian in the presence of the external vector, axial-vector, scalar and pseudoscalar fields, give estimates of its structure constants within the model [2]. Further on, following the ideology of the Chiral Perturbation Theory [3], we transform the bosonized chiral lagrangian into a phenomenological one and find relations (of Zweig rule type) between the corresponding structure constants. In the section 3 the $U(3)$-chiral bosonization in the presence of external $U_A(1)$-fields is considered in the heavy $\eta'$-meson mass limit. It makes possible to obtain the $SU(3)$ chiral lagrangian in the presence of external $U(3)$ fields and estimate new structure constants $L_{14}...L_{19}$. The corresponding vertices describe the heavy hadron decays which occur via the vacuum pseudoscalar channel. The generalized Zweig rules are found that allow to express the new structure coupling constants through the known ones. In the section 4 the conformal anomaly of the quark determinant [8] is used for the construction of the chiral lagrangian in external conformal-flat metric of the space-time. It turns out to be sufficient to determine three structure constants of the generalized chiral lagrangian in the vertices characterizing the meson energy-momentum tensor matrix elements. In the conclusion we discuss the correspondence of the predictions obtained by the low-energy QCD-bosonization method and to other theoretical and phenomenological estimates.

# 2 SU(3)-Chiral Lagrangian in the Low-Energy Bosonization Method.

Let us expose the scheme of the low-energy QCD bosonization for the $SU(3)$-chiral lagrangian. The QCD-bosonization is carried out by transition from the color quark and gluon fields to the collective boson variables describing the pseudogoldstone excitations which appear after the dynamical chiral symmetry breaking. For the generating functional of the singlet quark current correlators $Z(V,A,S,P)$ the bosonization is given by the following identity:

$$\int \mathcal{D}G\,\mathcal{D}\psi\mathcal{D}\bar\psi \exp(-S_{QCD}(\bar\psi,\,\psi,\,G;\;V,A,S,P)) = \int \mathcal{D}U \exp(-S_{eff}(U;\;V,A,S,P))\cdot\mathcal{R}, \quad (1)$$

where $V,A,S,P$ are, respectively, the external vector, axial-vector, scalar and pseudoscalar sources, and $U(x)$ is parametrized by pseudoscalar fields:

$$U = \exp(i\Pi(x)/F_\pi) \quad F_\pi \approx 93MeV. \quad (2)$$

In the right hand side of (1) the integration is made over the finite number of light boson states, and the heavier states' effects are included in the discrepancy functional $R$. The optimal bosonization at low energies guarantees $R \approx 1$. It corresponds to taking into account the total highest excitations' effects in the structure constants of chiral lagrangian. In the low-energy bosonization model [2] the dynamical chiral symmetry breaking in the low-energy region $\mathbf{L}$ is approximated by two parameters: $\Lambda$, the top spectrum boundary, and $M$, the spectrum asymmetry of the dynamical quarks. In this region $\mathbf{L}$= $(\Lambda, M)$ the choice of boson variables is based on the chiral noninvariance of the generating functional

under the local chiral transformations of external fields (chiral anomaly). The derivation of the effective chiral action is produced by integration of the generating functional over the group of local chiral rotations:

$$Z_{inv}^{-1} = \int \mathcal{D}U Z_{\psi}^{-1}(V^U, A^U, S^U, P^U), \tag{3}$$

where $Z_\psi$ is a quark part of the generating functional before integrating over gluons. The integration in (3) is carried out over the local $SU(3)$ group with invariant measure. Thus, we obtain a chiral invariant part of the generating functional $Z$. Due to this, one succeeds to calculate the chiral noninvariant part which is the effective action for the pseudoscalar fields, $S_{eff}(U; V, A, S, P)$:

$$Z = \int \mathcal{D}U \frac{Z}{Z_\psi(V^U, A^U, S^U, P^U)} << Z_{inv} >>_G \equiv \int \mathcal{D}U \exp(iS_{eff}(U; V, A, S, P)) << Z_{inv} >>_G, \tag{4}$$

where $<< ... >>_G$ means averaging over gluons.

In the model [2] $S_{eff}$ does not contain gluon fields, what is reflected in (4). The role of gluons is reduced only to the formation of the dimensional parameters of the theory, **L**$= (\Lambda, M)$, on the base of the equation of stability of the low-energy region , formed by gluon condensate [4].

In the low-energy region **L** the vertices of chiral lagrangian are classified by momentum and pseudoscalar meson mass degrees in accordance with the Chiral Perturbation Theory rules [3]:

$$S_{eff} = \int d^4x(\mathcal{L}_2 + \mathcal{L}_4) + S_{WZW}. \tag{5}$$

The Weinberg lagrangian of dimension 2, $\mathcal{L}_2$, looks as:

$$\mathcal{L}_2 = \frac{F_0^2}{4}[< (D_\mu U)^+ D^\mu U > + < \chi^+ U + U^+ \chi >], \tag{6}$$

where the symbol $< ... >$ means averaging over flavor indices, the external sources are included in the covariant derivative $D_\mu = \partial_\mu + [V_\mu, *] + \{A_\mu, *\}$ and in the complex density $\chi = 2B_0(S + iP)$ with the parameter $B_0$ connected with the condensate $< \bar{\psi}\psi >= -F_0^2 B_0$. In [2] these parameters are determined by model characteristics in the low-energy region L:

$$F_0^2 = \frac{N_c}{4\pi^2}(\Lambda^2 - M^2), \qquad F_0^2 B_0 = \frac{N_c}{2\pi^2}(\Lambda^2 M - \frac{1}{3}M^3). \tag{7}$$

The Wess-Zumino-Witten action, $S_{WZW}$, contains abnormal parity vertices, its form presented in [2], and in context of this paper it turns out to be important for the $U_A(1)$-bosonization (see the nest section).

The lagrangian of dimension 4, $\mathcal{L}_4$, responds for the fine structure of interaction of the pseudoscalar mesons and includes nine structure constants $I_k$:

$$\mathcal{L}_4^{eff} = I_1 < D_\mu U (D_\nu U)^\dagger D^\mu U (D^\nu U)^\dagger > + I_2 < D_\mu U (D_\mu U)^\dagger D^\nu U (D^\nu U)^\dagger >$$

$$
\begin{aligned}
&+I_3 < (D^2_\mu U)^\dagger D^2_\nu U > +I_4 < (D_\mu \chi)^\dagger D^\mu U + D_\mu \chi (D^\mu U)^\dagger > \\
&+I_5 < D_\mu U (D^\mu U)^\dagger (\chi U^\dagger + U \chi^\dagger) > +I_6 < U \chi^\dagger U \chi^\dagger + \chi U^\dagger \chi U^\dagger > \\
&+I_7 < \chi^\dagger U - U^\dagger \chi >^2 \\
&+I_8 < F^R_{\mu\nu} D^\mu U (D^\nu U)^\dagger + F^L_{\mu\nu} (D^\mu U)^\dagger D^\nu U > +I_9 < U^\dagger F^R_{\mu\nu} U F^{L\mu\nu} >, \qquad (8)
\end{aligned}
$$

where $F^L_{\mu\nu} = \partial_\mu L_\nu - \partial_\nu L_\mu + [L_\mu, L_\nu]$; $\quad L_\mu = V_\mu - A_\mu$; $\quad R_\mu = V_\mu + A_\mu$. After the bosonization one obtains the following estimates for the constants $I_k$:

$$
\begin{aligned}
&I_1 = \frac{N_c}{192\pi^2}, \qquad I_2 = -\frac{N_c}{96\pi^2}, \qquad I_3 = \frac{N_c}{96\pi^2}, \qquad I_4 = \frac{N_c M}{16\pi^2 B_0}, \qquad I_5 = 0, \\
&I_6 = -\frac{\Lambda^2 - M^2}{64\pi^2 B_0}, \qquad I_7 = 0, \qquad I_8 = \frac{N_c}{48\pi^2}, \qquad I_9 = \frac{N_c}{96\pi^2}. \qquad (9)
\end{aligned}
$$

From (9) one can see that the $SU(3)$-low-energy bosonization reproduces the coefficients $I_k$ in the main order of $N_c$ $\quad (I_k = O(N_c), \quad k \neq 7)$; the coefficient $I_7$ is saturated by the gluonic vacuum pseudoscalar configurations and is not determined correctly in the $SU(3)$-bosonization (see the next section).

The phenomenological chiral lagrangian can be found from the lagrangian (8) by rules of the Chiral Perturbation Theory, when the equation of motion for the lagrangian of dimension 2 is use:

$$
(D^2_\mu U)^\dagger U - U^\dagger D^2_\mu U - \chi^\dagger U + U^\dagger \chi = \frac{1}{3} < U^\dagger \chi - \chi^\dagger U >, \qquad (10)
$$

In the $SU(3)$ case, the following identities turn out to be useful:

$$
< A_\mu A_\nu A_\mu A_\nu >= -2 < A^2_\mu A^2_\nu > + \frac{1}{2} < A^2_\mu >^2 + < A_\mu A_\nu >^2 \qquad (11)
$$

The additional Gasser-Leutwyler lagrangian of dimension 4 [3] contains ten structure constants $L_i \quad (i = 1, ..., 10)$:

$$
\begin{aligned}
\mathcal{L}^{GL}_4 \;=\; & L_1 < (D_\mu U)^\dagger D^\mu U >^2 + L_2 < (D_\mu U)^\dagger D_\nu U >< (D^\mu U)^\dagger D^\nu U > \\
& + L_3 < (D^\mu U)^\dagger D_\mu U (D^\nu U)^\dagger D_\nu U > + L_4 < (D^\mu U)^\dagger D_\mu U >< \chi^\dagger U + U^\dagger \chi > \\
& + L_5 < (D^\mu U)^\dagger D_\mu U (\chi^\dagger U + U^\dagger \chi) > + L_6 < \chi^\dagger U + U^\dagger \chi >^2 \\
& + L_7 < \chi^\dagger U - U^\dagger \chi >^2 + L_8 < \chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger \chi > \\
& + L_9 < F^R_{\mu\nu} D^\mu U (D^\nu U)^\dagger + F^L_{\mu\nu} (D^\mu U)^\dagger D^\nu U > - L_{10} < U^\dagger F^R_{\mu\nu} U F^{L\mu\nu} >, \qquad (12)
\end{aligned}
$$

which turn out to be connected with the coefficients $I_k$ in the chiral bosonization models:

$$
\begin{aligned}
2L_1 &= L_2 = I_1, \quad L_3 = I_2 + I_3 - 2I_1, \quad L_4 = L_6 = 0, \quad L_5 = I_4 + I_5, \\
L_7 &= I_7 - \frac{1}{6} I_4 + \frac{1}{12} I_3, \quad L_8 = -\frac{1}{4} I_3 + \frac{1}{2} I_4 + I_6, \quad L_9 = I_8, \quad L_{10} = I_9. \qquad (13)
\end{aligned}
$$

The zero-valued $L_4, L_6$ are in compliance with the large-$N_c$ counting rules, since they give contribution of order $O(1) \quad (N_c \to \infty)$. This choice eliminates the uncertainty characteristic

for the $SU(3)$-lagrangians of dimensions 2 and 4, which appears due to the symmetry of observables under the one-parameter transformation:

$$\chi^{(\lambda)} = \chi + \frac{\lambda}{B_0^2}(\det \chi^+)\chi(\chi\chi^+)^{-1} \tag{14}$$

(Kaplan-Manohar symmetry). Here $\lambda$ is an arbitrary real number. Such a transformation changes values of the structure constants $L_6, L_7, L_8$ as follows:

$$L_6^\lambda = L_6 - \tilde{\lambda}, \qquad L_7^\lambda = L_7 - \tilde{\lambda}, \qquad L_8^\lambda = L_8 + 2\tilde{\lambda}, \tag{15}$$

where $\tilde{\lambda} = F_\pi^2\lambda/16B_0^2$. Setting $L_6 = 0$ in accordance with the Zweig rules, we evidently fix the constants $L_7, L_8$.

So far as the structure of the chiral lagrangian, presented in (8), is general for the QCD-bosonization models in the limit of large number of colors, the correlations (13) between the model coupling constants $I_k$ and the phenomenological ones $L_k$ characterize an uncertainty in fixing of the constants $I_k$ from the experimental data. There exists a two-parameter family of chiral bosonization models, giving the same values of observables.

# 3 $U_A(1)$ Generalization of the Chiral Lagrangian.

The chiral lagrangian in the presence of external fields, singlet in flavors, allows to conduct the bosonization of the singlet quark currents and pseudoscalar gluon density. To construct it correctly, it is necessary to extend the $SU(3)$-lagrangian up to the $U(3)$-type one and take into account correctly the influence of the vacuum QCD effects. The external $U_A(1)$-fields include the axial-vector singlet field $A_\mu$ and the source $\theta$ coupling with the pseudoscalar gluon density $G\widetilde{G}$:

$$\mathcal{L}_\theta = -\theta(x)G\widetilde{G}, \qquad G\widetilde{G} \equiv \frac{\alpha_s}{8\pi}G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a. \tag{16}$$

Let us accomplish the generalized scheme of the low-energy bosonization for the $U(3)$ case and consider the chiral fields $U(x) \longrightarrow \widetilde{U}(x)$, $\widetilde{U} = Uexp(i\eta_0/3)$ as collective variables. In general case, it gives the additional term in the lagrangian:

$$\mathcal{L}_-(\eta_0) = \Big(\eta_0 + i\xi < \chi^\dagger\widetilde{U} - \widetilde{U}^\dagger\chi >\Big)G\widetilde{G}, \tag{17}$$

which is conditioned by the chiral anomaly of the quark determinant. In accordance with the counting rules when $N_c$ is large, $\xi = O(1)$. When comparing the vertices (16) and (17) we see that the external source $\theta$ can be combined with the scalar density $\chi \longrightarrow \tilde{\chi} = \chi\exp(i\theta(x)/3)$, if change the variable $\eta_0 \longrightarrow \eta_0 - \theta$. After this, in the $SU(3)$ lagrangian there appear the new vertices with six phenomenological constants [5]:

$$\begin{aligned}
\mathcal{L}_4^{(D\theta)} &= iL_{14}D_\mu D^\mu\theta < \tilde{\chi}^\dagger U - U^\dagger\tilde{\chi} > + iL_{15}D_\mu\theta < (D^\mu\tilde{\chi})^\dagger U - U^\dagger(D^\mu\tilde{\chi}) > \\
&+ L_{16}D_\mu\theta D^\mu\theta < D_\nu U(D^\nu U)^\dagger > + L_{17}D_\mu\theta D_\nu\theta < D^\mu U(D^\nu U)^\dagger > \\
&+ L_{18}D_\mu\theta D^\mu\theta < \tilde{\chi}U^\dagger + U\tilde{\chi}^\dagger > + iL_{19}D_\mu\theta < U(D^\mu U)^\dagger D_\nu U(D^\nu U)^\dagger >,
\end{aligned} \tag{18}$$

which in the chiral bosonization models turn out to be connected with the structure constants of the $SU(3)$-lagrangian (8) and (12):

$$\begin{aligned} L_{15} &= -6L_{18} = -\frac{2}{3}(I_4 + I_5) = -\frac{2}{3}L_5; \\ L_{16} &= \frac{1}{2}L_{17} = -\frac{1}{6}L_{19} = \frac{2}{9}(I_1 + I_2 + I_3) = \frac{2}{9}(3L_2 + L_3) \end{aligned} \quad (19)$$

When forming the constant $L_{14}$, vacuum effects play a significant role. They are connected with that at large distance in QCD there appear infrared effects leading to non-zero value of the pseudoscalar gluon density correlator (topolgical susceptibility).

$$M_0^4 = -\int d^4x < 0|T(G\widetilde{G}(x)G\widetilde{G}(0))|0 >_0 . \quad (20)$$

This phenomenon guarantees solution of the so-called $U(1)$ problem and accounts for relatively large mass of the $\eta'$ meson. The averaging over gluons with taking into account (20) and with the multipole expantion approach used allows to calculate an additional contribution to the $U(3)$ meson lagrangian:

$$\widetilde{\mathcal{L}}(\eta_0) = -\frac{M_0^4}{2}\left(\eta_0 + i\xi < \chi^\dagger \widetilde{U} - \widetilde{U}^\dagger \chi >\right)^2 + O(\frac{1}{N_c}). \quad (21)$$

In this paper we are interested in the bosonization of the QCD-currents in the light pseudoscalar mesons sector. Thus, we consider the $\eta'$-meson mass as a large parameter in comparing to energies characteristic for the other meson's interaction. It gives us the reason to exclude the $\eta_0$-field by means of the large mass reduction method, i.e. the $1/M_0$ expansion and the Gaussian approach. As a result, we express the remaining constants $L_7, L_{14}$ via parameters characterizing interaction in the vacuum channel.

$$I_7 = -\frac{1}{2M_0^4}\left(\frac{F_0^2}{12} - \xi M_0^4\right)^2; \quad L_7 = -\frac{F_0^4}{288M_0^4} - \frac{1}{6}I_4 + \frac{1}{12}I_3 + \frac{\xi F_0^2}{12} - \frac{\xi^2 M_0^4}{2}. \quad (22)$$

$$L_{14} = \frac{F_0^4}{72M_0^4} - \frac{1}{3}I_4 - \frac{2}{3}I_5 - \frac{\xi F_0^2}{6} = -4L_7 + O(N_c), \quad (23)$$

The terms in these formulas are ordered according to their contributions at large-$N_c$, namely the first terms in $L_7, L_{14}$ are of order of $O(N_c^2)$, $N_c = 3$, and the next three ones are estimated as $O(N_c)$. In this case the Zweig rule works only numerically for $N_c$, and in the large-$N_c$ limit it is wrong, since the reduction of $\eta'$ by $1/M_0$ expansion is invalid. The saturation of the constants $L_7, L_{14}$ by terms of order $O(N_c)$ is in a good agreement with experimental estimations [5]:

$$L_7 = (-0.4 \pm 0.2) \cdot 10^{-3}, \qquad L_{14} = (2.3 \pm 1.1) \cdot 10^{-3}. \quad (24)$$

The constraints (13) and (19) presented above are characteristic for the most of the chiral bosonization models. In the chiral bosonization model, presented in the section 2, the parameter $\xi = 0$ that evidently allows to connect the remaining constants with the phenomenological ones, i.e. fix them from experiment. Thus, with the known values of

$\xi$ and $N_c$, the uncertainty in the model lagrangian vertices disappears. Thereby, so-called tachion vertices, which are proportional to $I_3$ and $I_4$, influence the $U_A(1)$-physics at low energies and can be estimated from experimental data.

# 4 Effective Chiral Lagrangian in the Conformal Metric and the Energy-Momentum Tensor.

The matrix elements of the energy-momentum tensor in the chiral theory are shown to be important for the description of the low-energy processes in which the scalar and tensor mesons are involved. Their calculation can be performed within the Chiral Perturbation Theory, when a nontrivial background metric in the chiral lagrangian is introduced. The metric tensor $g_{\mu\nu}$ is an external source after the variation of which the energy-momentum tensor can be found:

$$\frac{1}{2}\theta_{\mu\nu}(x) = \frac{\partial}{\partial g^{\mu\nu}(x)}\sqrt{-g}\mathcal{L}(\psi, A^{a\mu}, g_{\mu\nu})\Big|_{g_{\mu\nu}=\eta_{\mu\nu}}, \tag{25}$$

where $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$.

The effective lagrangian is built of the vertices invariant under the global coordinate transformations (diffeomorphisms). The general form of the chiral lagrangian can be written as follows:

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)}, \tag{26}$$

where $\mathcal{L}^{(2)}$, is still the Weinberg lagrangian generalized to an arbitrary metric. The $L^{(4)}$ includes terms of order of $p^4$:

$$\mathcal{L}^{(4)} = \mathcal{L}^{(4,g)} + \mathcal{L}^{(4,R)}. \tag{27}$$

The first part of $L^{(4,g)}$ in the formula (27) resembles in one's structure the lagrangian (12) in an arbitrary metric. The other part, $\mathcal{L}^{(4,R)}$, includes the curvature tensor:

$$\begin{aligned} \mathcal{L}^{(4,R)} &= L_{11} R < D_\mu U (D^\mu U)^\dagger > \\ &+ L_{12} R^{\mu\nu} < D_\mu U (D_\nu U)^\dagger > \\ &+ L_{13} R < \chi U^\dagger + U \chi^\dagger >, \end{aligned} \tag{28}$$

where $R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}$, $R = R_{\mu\nu}g^{\mu\nu}$, $R^\lambda_{\mu\sigma\nu} = \partial_\sigma \Gamma^\lambda_{\nu\mu} - \partial_\nu \Gamma^\lambda_{\sigma\mu} + \Gamma^\lambda_{\sigma\alpha}\Gamma^\alpha_{\nu\mu} - \Gamma^\lambda_{\nu\alpha}\Gamma^\alpha_{\sigma\mu}$. Thus, in an arbitrary metric three new phenomenological constants appear, which prove to be necessary for the energy-momentum tensor parametrization in the chiral theory

One can convince that for their estimates obtained by means of the low-energy bosonization method it is sufficient to consider the class of conformal metrics.

$$g_{\mu\nu} = \exp(-2\sigma)\eta_{\mu\nu}, \quad \eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1). \tag{29}$$

In this case the model lagrangian with conformal metric is obtaind from integration over the conformal and chiral anomalies, according to the bosonization scheme presented in the section 2.

In the conformal-flat metric the scalar curvature $R$ and the Ricci tensor $R_{\mu\nu}$ are functionals depending on $\sigma(x)$ and $\sigma_\mu \equiv \partial_\mu \sigma$:

$$\begin{aligned} R_{\mu\nu} &= \partial^2\sigma\eta_{\mu\nu} + 2\partial_\mu\partial_\nu\sigma - 2\sigma_\alpha\sigma^\alpha\eta_{\mu\nu} + 2\sigma_\mu\sigma_\nu \\ R &= -6e^{2\sigma}(\sigma_\mu\sigma^\mu - \partial^2\sigma), \end{aligned} \tag{30}$$

and the lagrange density, which contains new constants $(L_{11}, L_{12}, L_{13})$, takes the form:

$$\begin{aligned} & [-6(\sigma_\mu\sigma^\mu - \partial^2\sigma)L_{11} - (2\sigma_\mu\sigma^\mu - \partial^2\sigma)L_{12}] < D_\mu U(D^\mu U)^\dagger > \\ + \;& 2L_{12}(\sigma_{\mu\nu} + \sigma_\mu\sigma_\nu) < D^\mu U(D^\nu U)^\dagger > \\ - \;& 6L_{13}e^{-2\sigma}(\sigma_\mu\sigma^\mu - \partial^2\sigma) < \chi U^\dagger + U\chi^\dagger > \end{aligned} \tag{31}$$

The remaining part of the chiral lagrangian is easily reproduced from (12) by exchange of variables: $g_{\mu\nu} = e^{-2\sigma}\eta_{\mu\nu}$.

Let us proceed to the model evaluations of the structure constants $L_{11}, L_{12}, L_{13}$, which come from the low-energy chiral bosonization method. The generating functional in the presence of external metric can be reduced to the following form [10]:

$$Z_\psi = \int DqD\bar{q}\exp(i\int d^4x\,\bar{q}\exp(\sigma/2)\widetilde{\mathcal{D}}\exp(\sigma/2)q), \tag{32}$$

where the operator $\widetilde{\mathcal{D}} = \not{\mathcal{D}} + e^{-\sigma}(S + iP\gamma_5) \equiv \not{\mathcal{D}} + \widetilde{S} + i\widetilde{P}\gamma_5$, with $q \equiv e^{-2\sigma}\psi$. Thus, the conformal metric is induced by local dilatations of the Dirac operator $q \to e^{-\sigma/2}q$, and the corresponding effective action is calculated by integration over the conformal anomaly [8].

In order to find the chiral lagrangian in external metric we follow the low-energy QCD-bosonization scheme, presented in the paper [2]. The collective contribution of the conformal and chiral anomalies into the Wess-Zumino-type effective action has the following symbolic form:

$$W(\Pi, \sigma; \widetilde{S}, \widetilde{P}) = -\int_0^1 d\tau\,\mathrm{Tr}(\gamma_5\Pi < x|\mathcal{P}(\Lambda^2 - (\Phi_\tau\widetilde{\mathcal{D}}\Phi_\tau - iM)^2)|x>), \tag{33}$$

where $\Phi_\tau = \exp((\tau\sigma + i\gamma_5\Pi\tau)/2)$, and $\mathcal{P}(...)$ is the finite-mode projector [9] on the low-energy region in accordance to the model [4].

As the result of the chiral bosonization one has the effective action for pseudoscalar fields, which, in this case, is expressed by the difference of Wess-Zumino functionals in the external metric:

$$S_{eff}(\sigma, \Pi) = W(0, \sigma; \widetilde{S}, \widetilde{P}) - W(\Pi, \sigma; \widetilde{S}, \widetilde{P}). \tag{34}$$

It can be equivalently expressed by combination of the chiral lagrangians in conformal metric (see the section 2) and conformal ones in the presence of external pseudoscalar fields, namely:

$$\begin{aligned} & (W(0,0;\widetilde{S},\widetilde{P}) - W(0,\Pi;\widetilde{S},\widetilde{P})) \\ + & (W(0,\Pi;\widetilde{S},\widetilde{P}) - W(\sigma,\Pi;\widetilde{S},\widetilde{P})) \\ - & (W(0,0;\widetilde{S},\widetilde{P}) - W(\sigma,0;\widetilde{S},\widetilde{P})). \end{aligned} \tag{35}$$

Due to this, one succeeds to use the results [8], where the corresponding conformal effective action was calculated.

In order to convert the model lagrangian into phenomenological form (12) , we, as before, use the equation of motion:

$$U^\dagger D_\mu^2 U - (D_\mu^2 U)^\dagger U + 4\sigma_\mu (D_\mu U)^\dagger U + e^{-2\sigma}(\chi^\dagger U - U^\dagger \chi) = \frac{1}{3} e^{-2\sigma} < \chi^\dagger U - U^\dagger \chi >, \tag{36}$$

which, in this case, includes the dependence on the external metric, at that the tachion-like terms gives the additional contribution into the vertices $L_{11}, L_{12}, L_{13}$; in particular, the following vertex becomes essential:

$$\frac{N_c}{192\pi^2} < (D^2 U)^\dagger D^2 U > . \tag{37}$$

After the required transformations we come to the following model estimates of the structure constants:

$$\begin{aligned} L_{11} &= 1.58 \cdot 10^{-3} \\ L_{12} &= -3.2 \cdot 10^{-3} \\ L_{13} &= 0.3 \cdot 10^{-3}. \end{aligned} \tag{38}$$

One should point to the constraint $2L_{11} = -L_{12}$ which has a Zweig rule form and combines the vertices including curvature tensor in the conserving Einstein energy-momentum tensor:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R. \tag{39}$$

# 5 Conclusion.

In the paper we presented the extended low-energy bosonization of the vector, axial-vector, scalar and pseudoscalar $U(3)$-quark currents, and also the pseudoscalar gluon density and the quark energy-momentum tensor. We have used phenomenological solution of the $U(1)$-problem and a model of the low-energy region. On this way the estimations of the new structure constants $L_{11}, ..., L_{19}$ have been found and Zweig-type constraints on them were obtained.

It is of interest to compare our model estimations with experimental data, available at the moment, and the indirect estimates following from the hadron physics. As it was mentioned, the constraint $L_{14} = -4L_7$ is well justified by experimental information on the charmonium decays $\psi' \to J/\psi$ with emission of the $\pi$-or $\eta$-mesons [11].

Indeed, in the multipole expansion approach, the branching ratio of these reactions is given by:

$$\frac{\Gamma(\psi' \to J/\psi\pi^0)}{\Gamma(\psi' \to J/\psi\eta)} = \left|\frac{< 0|G\widetilde{G}|\pi^0 >}{< 0|G\widetilde{G}|\eta >}\right|^2 \frac{p_1^3}{p_2^3}. \qquad (40)$$

For adopted values of the current quark masses, it makes possible to find safely the constant $L_{14}$. Experimantal data are precise enough to evaluate $L_{14} = (2.3 \pm 1.1) \cdot\ 10^{-3}$.

For the description of the other structure constants the reliable experimental information is absent. However, some of them can be evaluated when proceeding from the phenomenological lagrangian, which includes heavier hadron states and is reduced to the chiral lagrangian in the infrared region [6]. In particular, the coefficients $L_{11}, L_{12}, L_{13}$ are saturated by the $\rho$-meson, tensor and scalar resonances and numerically must be of order:

$$\begin{aligned} L_{11} &= 1.6 \cdot 10^{-3} \\ L_{12} &= -2.7 \cdot 10^{-3} \\ L_{13} &= 0.9 \cdot 10^{-3}. \end{aligned} \qquad (41)$$

Here one can see a good agreement with evaluations of $L_{11}, L_{12}$ and the serious discrepancy of $L_{13}$. It is entailed by an uncertainty in modelling of the low-energy region and serves as a stimulus for proceeding development of the chiral bosonization method.

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